Stationary social learning in a changing environment

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Introduction

- Social learning literature:
  - two sources of information: private and social learning,
  - permanent state
- Changing state
  - natural assumption in many settings
  - rare and rapid political transitions: Arab Spring, 1830 liberal revolutions, carnival of Solidarity August 1980 - December 1981
- Why changing state matters?
  - Grossman-Stiglitz paradox makes stopping learning (i.e., informational cascade) not possible.

Question	Result
learning efficiency + welfare	no asymptotic learning, even with slowly changing state
is more social learning better?	it can be worse
behavior and beliefs	uniformity under slowly changing state, rare, rapid transitions

- Most striking results are when state is (very) persistent, but not permanent.
- Related lit: Moscarini Ottaviani Smith (98), Dasaratha Golub Hak (20), Kabos Meyer (21)

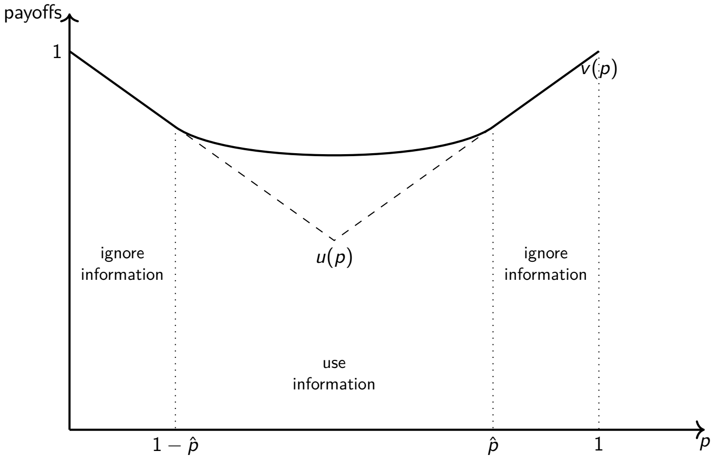
Model

- Markov-changing state $\theta_t \in \{0,1\}$.
    - $\lambda = P(\theta_{t+1} \neq \theta | \theta_t = \theta)$,
    - neither independent ($\lambda = \frac{1}{2}$), nor permanent ($\lambda = 0$),
    - slow transitions ($\lambda \to 0$)
- Social learning: in each period, continuum of short-lived agents
    - random sample of n actions from the previous period(s),
    - private signal at cost $c \geq 0$,
    - action $a \in \{0,1\}$,
    - utility $u(a, \theta_t) = \mathbf{1}(a = \theta_t)$.
- Stationary equilibrium $\mu \in \Delta(X \times \{0,1\})$, where
    - $x \in X = [0,1]$ if the fraction of population playing 1.

Model

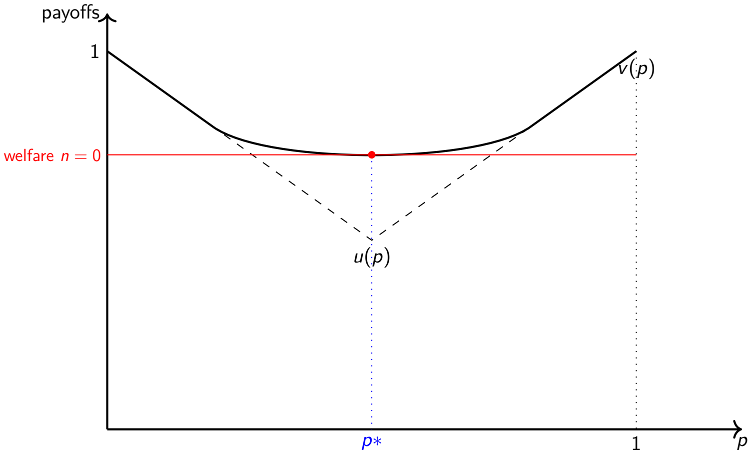
- Assumption: private signal is
  - either costly $c > 0$, or
  - free but with bounded precision,
- p - private belief about the state
  - $u(p)$ - expected payoff from optimal action
  - $v(p) \geq u(p)$ - expected payoff from optimally using information and then taking action

Model



- Assumption implies that $v(p) = u(p)$ for extreme beliefs.

Welfare $n = 0$



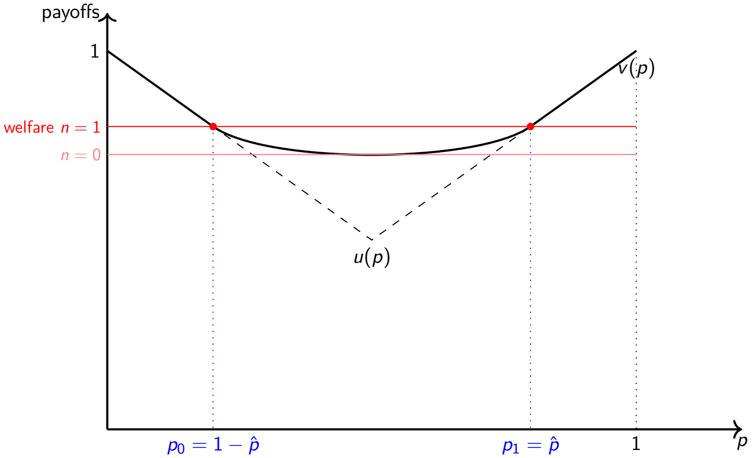
- each generation is identical and can only learn from private signal

Welfare

Permanent state

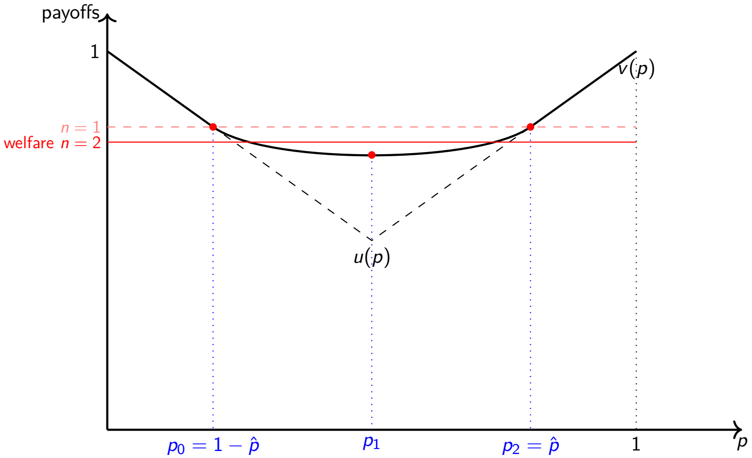
- Permanent state ($\lambda = 0$)
  - $n = 0$: no social learning
  - $n = 1$: some social learning but herding (Banerjee 92, Bikchandani et al 92)
  - $n = 2$: asymptotic full learning (Banerjee Fudenberg 2004)
- If state is permanent, social learning helps!

Welfare $n = 1$



- $p_0, p_1 \in [1 - \hat{p}, \hat{p}]$ and if $\lambda \leq \lambda^*$, then $p_1 = \hat{p} = 1 - p_0$.
  - p_k - belief after observing k agents (out of n sample) playing 1.

Welfare $n = 2$



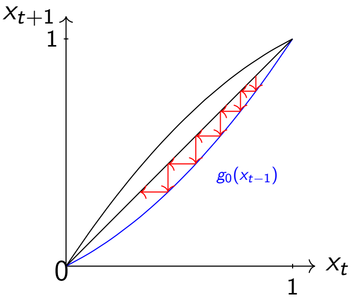
- **Theorem**: $p_0, p_1, p_2 \in [1 - \hat{p}, \hat{p}]$ and if $\lambda \leq \lambda^*$, then $p_2 = \hat{p} = 1 - p_0$.

Welfare $n = 2$

Proof sketch

- Suppose $p_2 > \hat{p}$: non-confused agents *don't* buy information.
  - ϕ_θ the probability that an agent with interim beliefs $p_1 = \frac{1}{2}$ (who thus acquires info) plays action 1 in state θ.
  - x_t the fraction of agents playing action 1 at date t

$$x_{t+1} = x_t^2 + 2x_t(1 - x_t)\phi_{\theta_{t+1}} =: g_{\theta_t}(x_t) .$$



Welfare $n = 2$

Proof sketch

$$x_{t+1} = x_t^2 + 2x_t(1 - x_t)\phi_{\theta_{t+1}} =: g_{\theta_t}(x_t).$$

- Sooner or later, x_t will be close to 0.
- Around 0, geometric random walk with Markov drift.

$$x_{t+1} \simeq \left(2\phi_{\theta_{t+1}}\right) x_t$$

  - since $(2\phi_0)(2\phi_1) = 4\phi_0(1 - \phi_0) < 1$, the drift is negative.

$\Rightarrow$ $\lim x_t \in \{0, 1\}$, a.s.

$\Rightarrow$ Samples are uninformative

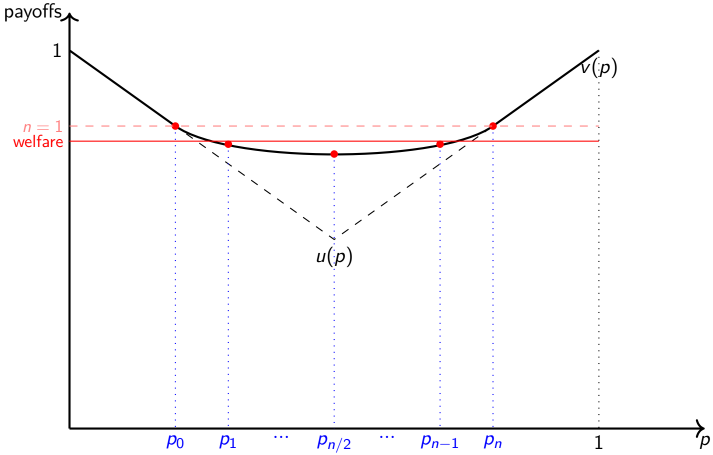
Welfare $n \geq 2$

- For $n > 2$, we need some assumptions:
  - persistent (but not permanent) state: $\lambda \to 0$
  - precise signals: $n^2 \phi_1 (1 - \phi_1) < 1$,
    - example: perfect signals
  - regular equilibrium: $p_{n-1} \geq \frac{1}{2}$.

Theorem

Under the above assumptions, in equilibrium, $\lim_{\lambda \to 0} p_n^\lambda = \hat{p} = 1 - \lim p_0^\lambda$.

Welfare $n \geq 2$



Behavior: Consensus

- Let $\mu_\lambda \in \Delta(X \times \{0,1\})$ be the stationary equilibrium.

Theorem

If $n \geq 2$, then, for each $\varepsilon > 0$

$$\lim_{\lambda \to 0} \mu_\lambda \{\varepsilon \leq x \leq 1 - \varepsilon\} = 0.$$

- uniform behavior, most of the time,
- together with previous result, uniform beliefs

Behavior: Consensus

Proof sketch

- Each agent observes random sample $(a_1, ..., a_n)$
  - a_1 is λ-close to optimal action,
  - a_2 cannot add too much information if λ is small,

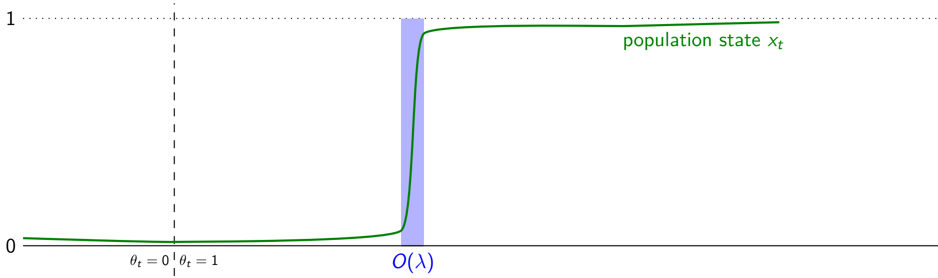
  $\Rightarrow$ $a_1 = a_2$, most likely.

Theorem

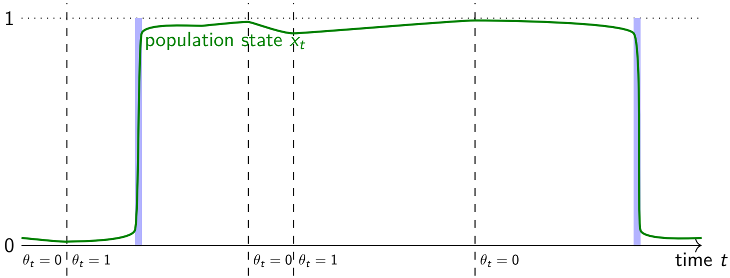
There exists a constant $K < \infty$ such that

$$\int x\,(1-x)\,d\mu^{\lambda}\,(x,\theta) \leq K\lambda.$$

Behavior: Rapid transitions

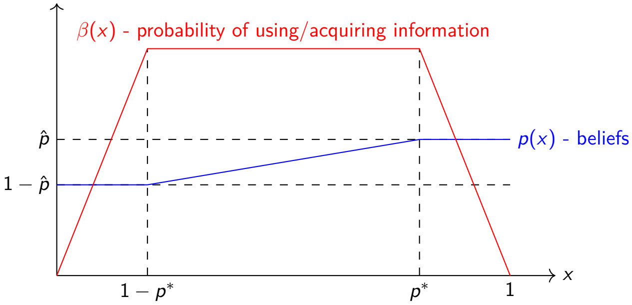


Behavior: Rapid transitions



Comments: $n \to \infty$

- Continuum time version <- we can compute equilibria.
- Binary signals



Comments: continuum actions

- Suppose $n \geq 2$, and
  - $A = [0, 1]$ and $u(a, \theta) = -(a - \theta)^2$,
  - perfect signals
- With permanent state ($\lambda = 0$) -> asymptotic learning and welfare $=$ 1.
- With persistent state ($\lambda > 0$) -> there are stationary equilibria with welfare $<$1.

Conclusions

- Social learning with changing state
- Even when state is very persistent (but not permanent):
    - no asymptotic learning, uniformly bounded welfare
- Additionally, when state is persistent
    - behavior and beliefs exhibit consensus,
    - rare and rapid transitions.