Bargaining with Mechanisms and Two-Sided Incomplete Information

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Introduction

- Bargaining with sophisticated offers in real world
  - menus,
  - menus of menus ("I divide, you choose"),
  - mediation, arbitration (example: "trial by gods"),
  - change in bargaining protocols,
  - deadlines or delays, etc.
- Previous work - one-sided incomplete information.
- Here,
  - non-cooperative random-proposer bargaining, where
  - players offer mechanisms to find a resolution, and with
  - two-sided incomplete information.

Introduction

Results

- Tools to solve such models.
- Main results for single good + transfers environment
    - two (private value) types for each player,
- Results:
    - non-trivial payoff bounds that depend on the bargaining power,
    - "unique" payoffs for "large" subspace of initial beliefs.

Model

Bargaining game

- Two players $i = 1, 2$,
  - sometimes third player (“mediator”).
- Bargaining game
  - multiple rounds until offer is accepted, discounting $\delta < 1$,
  - random proposer: Player i is a proposer with probability β_i, where $\beta_1 + \beta_2 = 1$,
    - includes single-proposer games $\beta_i \in \{0, 1\}$,
  - proposer proposes a mechanism: *a static or finite-horizon game with outcomes in the outcome space,*
  - once the offer is accepted, it is implemented (the mechanism game is played) and the bargaining game ends.
- Perfect Bayesian Equilibrium:
  - no updating beliefs about player i after $-i$'s action.
  - correlation device.

Model

Environment: Single good + transfers

- Environment: single good plus transfers:
  - types: valuations $v \in \mathbb{R}$,
  - preferences: $vq - \tau$,
  - single good $q_1 + q_2 = 1$, $q_i \geq 0$,
  - transfers: $\tau_1 + \tau_2 = 0$,
- Two types for each player $T_i = \{l_i, h_i\}$

$$0 \leq l_1 \leq l_2 < h_1 \leq h_2,$$

  - p_i - probability of type h_i

Model

Mechanisms

- Each offer is a *mechanism*:
  - a (static or extensive-form) finite or "compact" game G.
  - examples: single-offers, menu, menu of menus, auctions, etc.
- No revelation principle.
- Equilibrium payoffs in mechanism G given beliefs p: $m_G(p) \subseteq R^{T_1 \cup T_2}$
  - payoff vector $u \in R^{T_1 \cup T_2}$ where $u(t_i) \in R$,
- Equilibrium correspondence $m_G : \Delta T \rightrightarrows R^{T_1 \cup T_2}$.
- For each "compact" game, m_G is a "Kakutani correspondence": u.h.c, non-empty-valued, and convex (due to public correlation).

Model

Incentive compatible allocations

- Given beliefs p, allocation $q_i(.), \tau(.)$ is incentive compatible iff

$$\text{standardincentive constraintsfor each } t_i, s_i$$

- Payoffs in incentive compatible allocation given p

$$u_i(t_i|q,\tau) = \sum_{t_{-i}} p(t_{-i})(t_i q_i(t_i, t_{-i}) - \tau_i(t_i, t_{-i})).$$

- IC correspondence:

$$U(p) = \{u(.|q,\tau) : \xi \text{ is IC given } p\} \subseteq R^{T_1 \cup T_2}.$$

- For each mechanism G, $m_G \subseteq U$.
  - the geometry of the correspondence $U(.)$ is the true "parameter" of the model.

Model

Mechanisms

- Abstract mechanism: m is Kakutani correspondence st. $m \subseteq U$.
- "Implementation Theorem": does each abstract mechanism have a game that makes it a "real" mechanism?
  - likely not true,
  - true "approximately": under virtual implementation conditions (Abreu-Matsushima),
  - this is why we need a mediator.

Model

Derived games

- Given a mechanism m or set of mechanisms A, construct new game:
  - $MM_i(A)$ - menu of mechanisms for player i,
  - $IP_i(m)$ - informed principal problem of player i with player $-i$ outside option m,

$$IP_i(m) = MM_i(\{MM_{-i}(n, m) : n \text{ is a mechanism}\})$$

  - $\alpha \in \Delta A$- randomly chosen mechanism,
- Bargaining game:

$$B = (IP_1(\delta B))^{\beta_1}(IP_2(\delta B))^{\beta_1}$$

Model

Commitment

- Players are not committed to future offers.
- Players are committed to implementing a mechanism once offered and accepted:
    - hence, less commitment than, say in the *limited commitment* literature (V. Skreta, L. Doval).
- Comments:
    - what the “lack of commitment” means in my setting?
        - how to bargain about deadlines if we are not really committed to them)
    - “lack of commitment” is a restriction on the space of mechanisms,
    - commitment is not necessarily helpful to the agent who can exercise it.

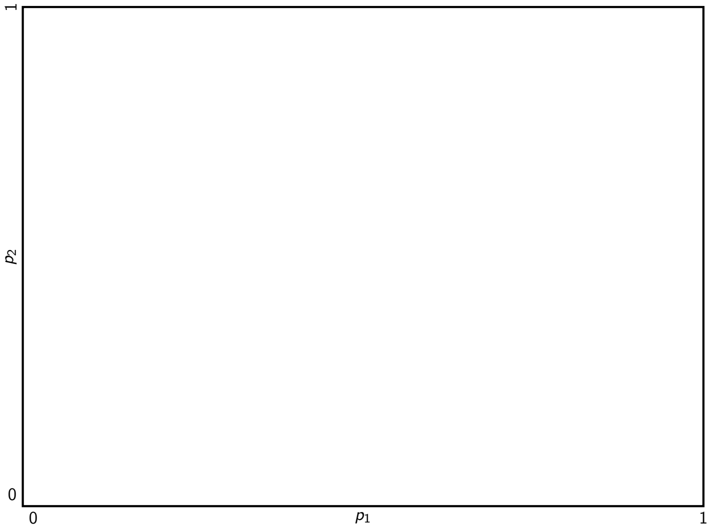
Results

Benchmarks

- Complete information: players split the higher payoff in fractions β and $1-\beta$.
- One-sided incomplete Peski (22):
  - the equilibrium payoffs are unique,
  - In an equilibrium, random property rights (RPR) mechanism is offered:
  - agent *i* gets the good with probability β_i,
  - if so, she can make a single take-it-or-leave-it <u>sell</u> offer,
  - regardless if the offer is accepted or not, the mechanism ends.
- Two sided incomplete information:

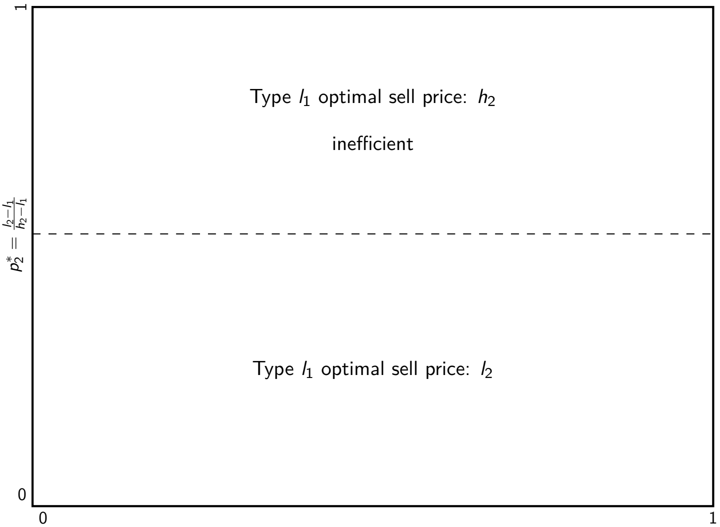
Results

Beliefs space



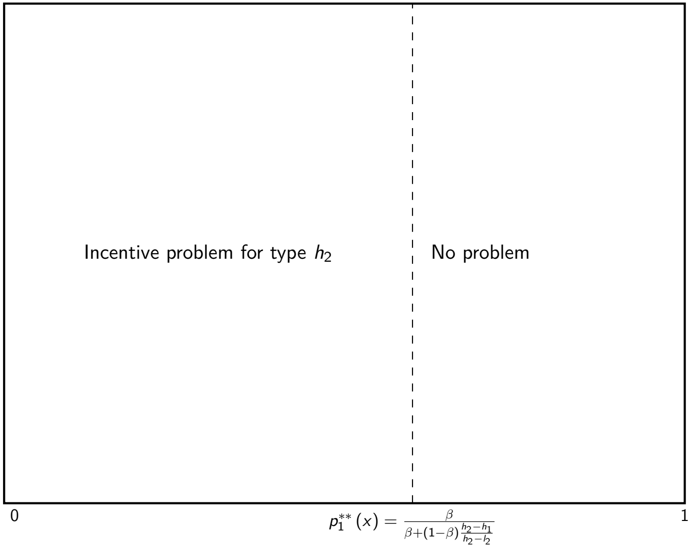
Results

Beliefs space + incentive constraint for pl 1



Results

Beliefs space + incentive constraint for pl 2



Incentive problem for type h_2

No problem

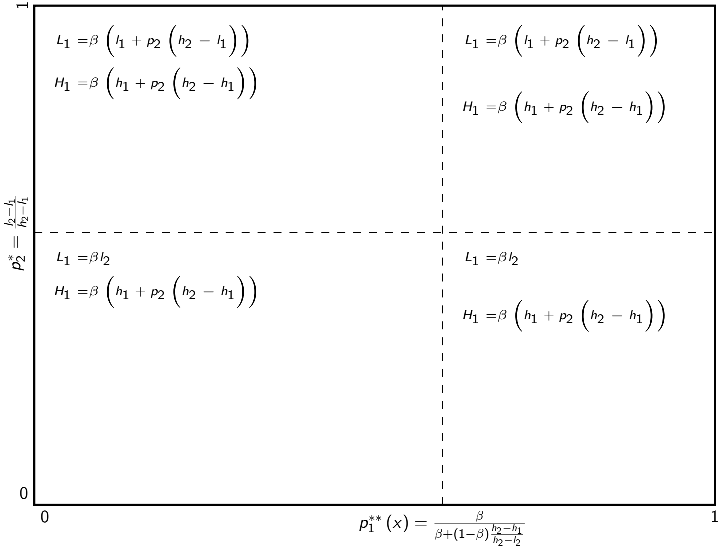
0

$p_1^{**}(x) = \frac{\beta}{\beta+(1-\beta)\frac{h_2-h_1}{h_2-l_2}}$

1

Results

Random property rights payoffs player 1



$L_1 = \beta\left(l_1 + p_2\left(h_2 - l_1\right)\right)$

$H_1 = \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right)$

$L_1 = \beta\left(l_1 + p_2\left(h_2 - l_1\right)\right)$

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$L_1 = \beta l_2$

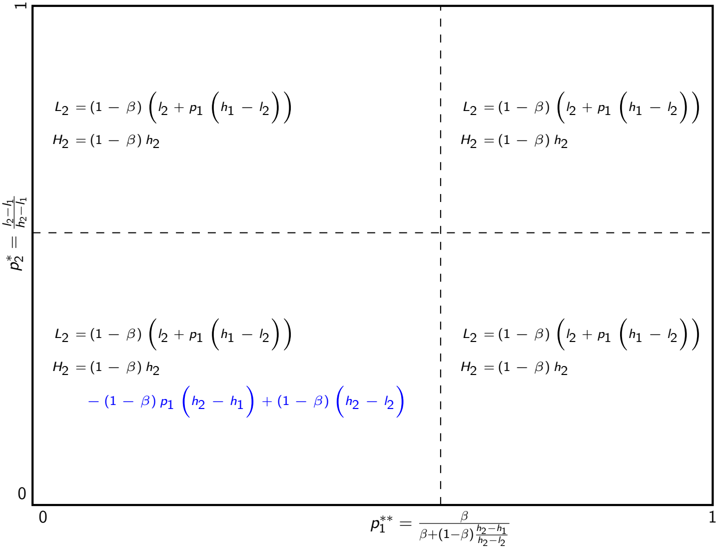
$H_1 = \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right)$

$L_1 = \beta l_2$

$H_1 = \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right)$

Results

Random property rights payoffs: player 2



$p_2^* = \frac{l_2 - l_1}{h_2 - l_1}$

$p_1^{**} = \frac{\beta}{\beta + (1-\beta)\frac{h_2 - h_1}{h_2 - l_2}}$

$$L_2 = (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right)$$
$$H_2 = (1-\beta)\,h_2$$

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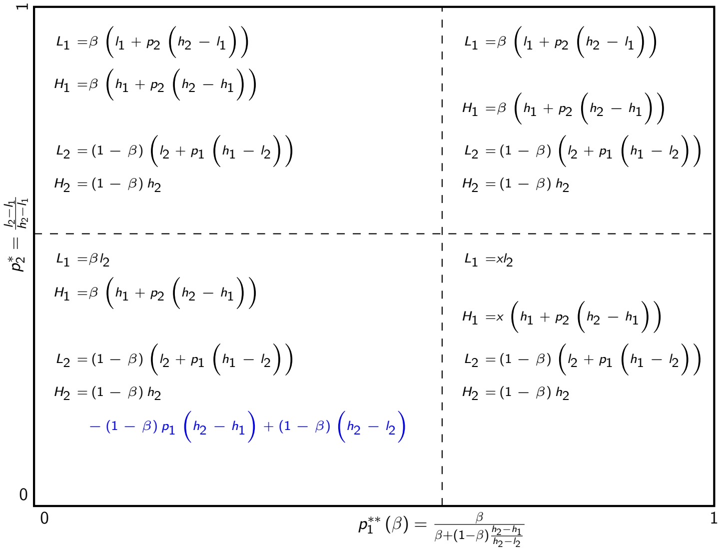
$$L_2 = (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right)$$
$$H_2 = (1-\beta)\,h_2 - (1-\beta)\,p_1\left(h_2 - h_1\right) + (1-\beta)\left(h_2 - l_2\right)$$

$$L_2 = (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right)$$
$$H_2 = (1-\beta)\,h_2$$

Results

Random property rights payoffs: both players



$$L_1 = \beta\left(l_1 + p_2\left(h_2 - l_1\right)\right)$$
$$H_1 = \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right)$$
$$L_2 = (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right)$$
$$H_2 = (1-\beta)\,h_2$$

$$L_1 = \beta\left(l_1 + p_2\left(h_2 - l_1\right)\right)$$
$$H_1 = \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right)$$
$$L_2 = (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right)$$
$$H_2 = (1-\beta)\,h_2$$

$$L_1 = \beta l_2$$
$$H_1 = \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right)$$
$$L_2 = (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right)$$
$$H_2 = (1-\beta)\,h_2$$
$$-(1-\beta)\,p_1\left(h_2 - h_1\right) + (1-\beta)\left(h_2 - l_2\right)$$

$$L_1 = x l_2$$
$$H_1 = x\left(h_1 + p_2\left(h_2 - h_1\right)\right)$$
$$L_2 = (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right)$$
$$H_2 = (1-\beta)\,h_2$$

Results

Theorem

In any equilibrium, each type of each player gets at least its random property rights payoffs.

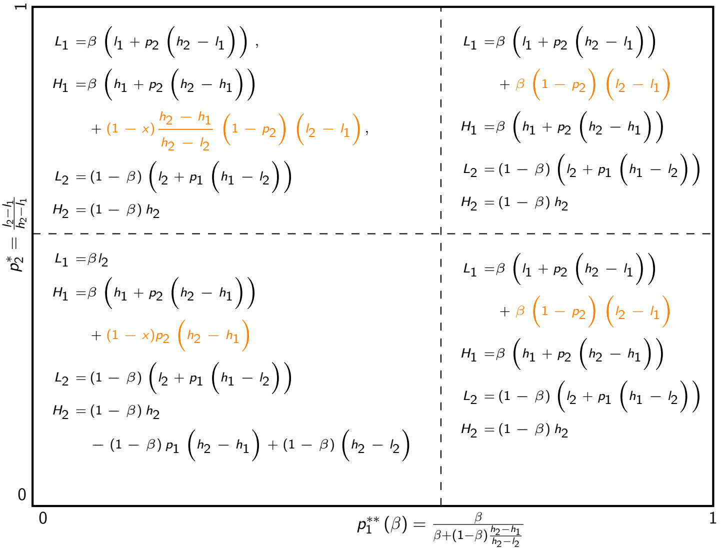
- Intuition:
- equilibrium payoffs become unique when:
  - $l_2 \to l_1$, or
  - $h_2 \to h_1$, or
  - (one sided offer)$\beta_1 \to 0$, or $\beta_1 \to 1$.
- In general, bounds are not tight.

Results

- In general, bounds are not tight.
- The reason is that RPR payoffs are not interim efficient.

Results

Interim efficient payoffs: player 1 gets all the surplus



$$
\begin{aligned}
L_1 &= \beta \left(l_1 + p_2 \left(h_2 - l_1 \right) \right), \\
H_1 &= \beta \left(h_1 + p_2 \left(h_2 - h_1 \right) \right) \\
&\quad + (1 - x) \frac{h_2 - h_1}{h_2 - l_2} \left(1 - p_2 \right) \left(l_2 - l_1 \right), \\
L_2 &= (1 - \beta) \left(l_2 + p_1 \left(h_1 - l_2 \right) \right) \\
H_2 &= (1 - \beta)\, h_2
\end{aligned}
$$

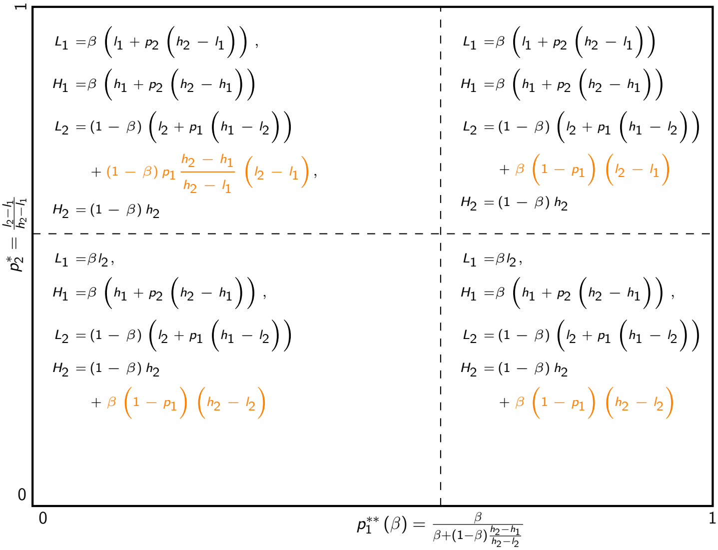
$$
\begin{aligned}
L_1 &= \beta \left(l_1 + p_2 \left(h_2 - l_1 \right) \right) \\
&\quad + \beta \left(1 - p_2 \right) \left(l_2 - l_1 \right) \\
H_1 &= \beta \left(h_1 + p_2 \left(h_2 - h_1 \right) \right) \\
L_2 &= (1 - \beta) \left(l_2 + p_1 \left(h_1 - l_2 \right) \right) \\
H_2 &= (1 - \beta)\, h_2
\end{aligned}
$$

$$
\begin{aligned}
L_1 &= \beta l_2 \\
H_1 &= \beta \left(h_1 + p_2 \left(h_2 - h_1 \right) \right) \\
&\quad + (1 - x) p_2 \left(h_2 - h_1 \right) \\
L_2 &= (1 - \beta) \left(l_2 + p_1 \left(h_1 - l_2 \right) \right) \\
H_2 &= (1 - \beta)\, h_2 \\
&\quad - (1 - \beta)\, p_1 \left(h_2 - h_1 \right) + (1 - \beta) \left(h_2 - l_2 \right)
\end{aligned}
$$

$$
\begin{aligned}
L_1 &= \beta \left(l_1 + p_2 \left(h_2 - l_1 \right) \right) \\
&\quad + \beta \left(1 - p_2 \right) \left(l_2 - l_1 \right) \\
H_1 &= \beta \left(h_1 + p_2 \left(h_2 - h_1 \right) \right) \\
L_2 &= (1 - \beta) \left(l_2 + p_1 \left(h_1 - l_2 \right) \right) \\
H_2 &= (1 - \beta)\, h_2
\end{aligned}
$$

Results

Interim efficient payoffs: player 2 gets all the surplus



$$\begin{aligned} L_1 &= \beta\left(l_1 + p_2\left(h_2 - l_1\right)\right), \\ H_1 &= \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right) \\ L_2 &= (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right) \\ &\quad + (1-\beta)\, p_1 \frac{h_2 - h_1}{h_2 - l_1}\left(l_2 - l_1\right), \\ H_2 &= (1-\beta)\, h_2 \end{aligned}$$

$$\begin{aligned} L_1 &= \beta\left(l_1 + p_2\left(h_2 - l_1\right)\right) \\ H_1 &= \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right) \\ L_2 &= (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right) \\ &\quad + \beta\left(1 - p_1\right)\left(l_2 - l_1\right) \\ H_2 &= (1-\beta)\, h_2 \end{aligned}$$

$$\begin{aligned} L_1 &= \beta l_2, \\ H_1 &= \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right), \\ L_2 &= (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right) \\ H_2 &= (1-\beta)\, h_2 \\ &\quad + \beta\left(1 - p_1\right)\left(h_2 - l_2\right) \end{aligned}$$

$$\begin{aligned} L_1 &= \beta l_2, \\ H_1 &= \beta\left(h_1 + p_2\left(h_2 - h_1\right)\right), \\ L_2 &= (1-\beta)\left(l_2 + p_1\left(h_1 - l_2\right)\right) \\ H_2 &= (1-\beta)\, h_2 \\ &\quad + \beta\left(1 - p_1\right)\left(h_2 - l_2\right) \end{aligned}$$

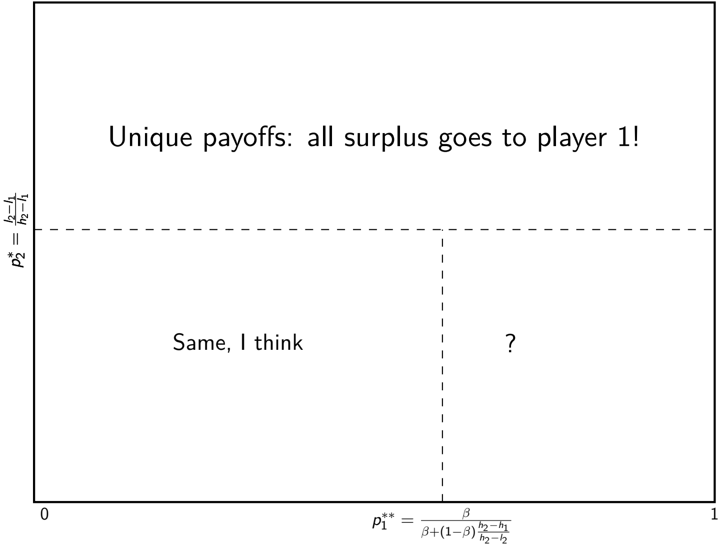
Results

Theorem

As $\delta \to 1$, when $p_2 > p_2^*$, the equilibrium payoffs are interim efficient and maximize the expected player 1 payoffs subject to the constraint that player 2 receives their RPR payoffs.

- player 2 payoffs are unique (for each type separately)
- player 1 expected payoffs are unique and subject to RPR bounds (and IC constraints).
- Idea of the argument: construct mechanisms that cannot be rejected.

Results



Unique payoffs: all surplus goes to player 1!

Same, I think

?

$p_2^* = \frac{l_2 - l_1}{h_2 - l_1}$

0

$p_1^{**} = \frac{\beta}{\beta + (1-\beta)\frac{h_2 - h_1}{h_2 - l_2}}$

1