Bargaining with Mechanisms

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Introduction

- Sophisticated offers in real world
  - menus,
  - menus of menus (“I divide, you choose”),
  - deadlines or delays,
  - negotiation chapters,
  - propose arbitration (example: trial by gods), propose a change to bargaining protocols, etc.

Introduction

- Model of bargaining, where players offer mechanisms to find a resolution.
- Why mechanisms help?
  - screening: which type of the opponent wants what?
  - signaling: how to protect oneself from revealing information?
  - "belief threats": can opponent's adversarial beliefs be tested?

Model

Environment

- Alice (informed) and Bob (uninformed):
  - Bob's beliefs F about Alice's preferences $u \in [0,1]$,
  - Bob's preferences $v \in [0,1]$ are known.
- Single good + transfers,
  - Alice's utility: $qu + t$
  - Bob's utility $(1-q)\,v - t$
- Bargaining game
  - multiple rounds until offer is accepted, discounting $\delta < 1$,
  - once the offer is accepted, it is implemented and the game ends,
  - random proposer: Alice is a proposer with i.i.d. probability $\beta = \beta_A$ and Bob with prob. $1-\beta = \beta_B$,
    - both sides may make offer,
    - includes single-proposer games $\beta \in \{0,1\}$.

Model

Mechanisms as offers

- Each offer is a *mechanism*: a finite-horizon extensive-form game.
  - $m = \left(\left(S_A^t, S_B^t \right)_{t \leq T}, \chi \right)$
  - allocation: $\chi : \prod_{i,t} S_{i,t} \to X$,
  - $T < \infty$ and S_i^t compact.
- Examples: single-offers, menu, menu of menus
- When an offer is accepted, mechanism is implemented, and the game ends.
- Main result hold as long as $\mathcal{M}$ contains menus and menus of menus.

Model

Equilibrium

- “Perfect Bayesian Equilibrium,”
  - existence is an issue (assume cheap talk and randomization device for this),
  - we show the existence of $\mathcal{M}$ is “compact”,
  - menus + menus of menus is “compact”.

Model

Mechanisms as offers

- For each mechanism m, only equilibrium payoffs matter:
  - payoffs
  - define equilibrium payoff correspondence $E(m) : \Delta[0,1] \rightrightarrows \mathbb{R}^{[0,1]} \times \mathbb{R}$.
- Mechanisms are Kakutani if $E(m)$ is u.h.c., convex- and non-empty-valued,
- $\mathcal{M}$ is "compact" if
  - all mechanisms are Kakutani and
  - the correspondence $E : \mathcal{M} \times \Delta[0,1] \rightrightarrows \mathbb{R}^{[0,1]} \times \mathbb{R}$ is u.h.c.
- **Assumption**: $\mathcal{M}$ is compact.

Model

Equilibrium notion and existence

- Menu of mechanism game:
  - makes an announcement $a \in A$,
  - chooses from the compact set of mechanism $\mathcal{M}$,
  - after which public randomization is observed and one of the continuation payoffs is implemented:
- PBE:
  - strategy: $\alpha \in \Delta(\mathcal{M} \times A)$,
  - posteriors: $p : \mathcal{M} \times \mathcal{A} \to$ "beliefs",
  - continuations $v : \mathcal{M} \times A \to \Delta$("payoffs"),
  - p and v are measurable.
- Bargaining game - a sequence of menu of mechanisms games,
  - PBE of bargaining game = sequence of PBEs in the menu of mechanism games, where continuation payoffs in chosen mechanisms are PBEs of the subsequent games.

Main result

Complete information

- Complete information bargaining: Alice u, and Bob v (fixed).
- Surplus $\max(u, v)$.
- Both players split the surplus, and receive

$$(\beta \max(u, v), (1 - \beta) \max(u, v))$$

  - the player with higher utility gets the good and pays out a fraction of its value in the form of a transfer.
- This is not incentive compatible if Alice's utility $u > v$.

Main result

Optimal mechanisms

- Alice's optimal (ICR) mechanism:
  - own the good and offer it for sale at price v,
  - payoffs: $(\max(u, v), 0)$.
- Bob's optimal mechanism:
  - own the good and offer it for sale at price $p^* \in \arg\max vF(p) + p(1 - F(p))$
  - payoffs
$$(\max(u - p^*, 0), vF(p^*) + p^*(1 - F(p^*))).$$
  - Assume for simplicity that p^* is unique.

Main result

Theorem

Suppose $\mathcal{M}$ contains all menus and menus of menus. Then, in the unique equilibrium, the expected payoffs are as if

- *with prob β, Alice implements her optimal mechanism,*
- *with prob. $1-\beta$, Bob implements his optimal mechanism.*

- $\beta-$random property ("usage" + "sell") right

Main result

- "Incentive-efficient", but not ex post efficient:
  - Alice types $v < u < p^*$ do not get the good with prob. $1 - \beta$,
- Bob's payoffs are continuous and convex in F,
- Myerson's neutral solution,
- Bob's constrained commitment: the outcome is best for Bob subject to Alice receiving her complete information payoffs.

Proof

- Equilibrium “construction”.
- Lower bound on Bob’s payoffs.
- Lower bound on Alice’s payoffs.

Proof

Preliminaries

- For each α, let $m_{\alpha}^{*}(F)$ be the best mechanism for Bob st. Alice receives her complete info payoffs $y_{\alpha}(u) = \alpha \max(u, v)$.
  - α-random property rights, or
  - 3-element Alice's menu.
- Menu Y_{α,p^*}:
  - Bob gets the good and Alice receives transfer αv,
  - Alice gets the good with prob. α,
  - Alice gets the good, and pays $(1-\alpha)\, p^*$,
- Payoffs are affine in α,

$$\text{Alice payoffs: } y_{\alpha}^{*}(u; F) := \alpha \max(u, v) + (1-\alpha)\max(u - p^*, 0)$$

$$\text{Bob payoffs: } \Pi_{\alpha}^{*}(F) := (1-\alpha)\left[vF(p^*) + p^*(1 - F(p^*))\right],$$

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Proof

Equilibrium

- In equilibrium, if player i is chosen a proposer, they offer $m^*_{\alpha_i}$, where

$$\alpha_A = 1 - \delta\left(1 - \beta\right) \text{ and } \alpha_B = \delta\beta,$$

  - the average payoff is y^*_β and Π^*_β, where $\beta = \beta\alpha_A + \left(1 - \beta\right)\alpha_B$,
  - Bob is indifferent between accepting Alice's offer and waiting, $\Pi^*_{\alpha^\alpha} = \Pi^*_{1-\delta(1-\beta)} = \delta\Pi^*_\beta$,
  - Alice (weakly) prefers to accept Bob's offer than to wait, $y^*_{\alpha^B} = y^*_{\delta\beta} \geq \delta y^*_\beta$.

Proof

Equilibrium

- What if players make out-of-equilibrium offers?
- If Bob deviates:
  - each type of Alice optimally accepts or rejects,
  - the lower bound on rejection payoffs are Alice's (α^B-) complete info payoffs ,
  - but because $m^*_{\alpha^B}$ is Bob-optimal, Bob cannot profit from the deviation.
- If Alice offers $m \neq m^*_{\alpha^A}$: there is Bob's belief F_m st.
  - either Alice's mechanism has an equilibrium with payoffs $\leq y_{\alpha^A}(u)$ for all her types (so, not a profitable deviation), or
  - Bob's payoffs are $\leq \Pi_{\alpha^A}(F_m)$, and Bob prefers to wait.

Proof

Lower bound on Bob's payoffs

- Can Bob get a lower payoff? No.
- Suppose $x > \beta$ is the highest possible so that $\Pi_x^*(G)$ is the tight lower bound on Bob's eq. payoff across any beliefs G,
  - then, there is always Alice's type (in the support of G) that receives $\leq y_x$ in any equilibrium.
- In equilibrium with payoffs $\Pi_x^*(G)$:
  - Bob's counteroffer $m_{\delta x+\varepsilon}^*(G)$.
  - If accepted, this is strictly profitable (for small ε) for Bob, $\Pi_x^*(G) < \beta_B \Pi_{\delta x+\varepsilon}^*(G) + (1-\beta_\beta)\,\delta\Pi_x^*(G)$
  - The offer will be accepted.
    - Alice payoffs from accepting $y_{\delta x+\varepsilon}^* > y_{\delta x}^* \geq \delta y_x$,
    - If rejected, posterior beliefs G'.
    - Because Bob receives $\Pi_x(G')$ in the continuation, at least one G'-positive prob. Alice's type must get y_x (somewhere the constraint must be binding).
    - But then, this type should not have rejected - hence beliefs are not G'.

Proof

Lower bound on Alice's payoffs

- Can Alice's get a lower payoff? Not less than y^*_β.
- Suppose $x \leq 1$ is the lowest possible so that Alice's type u receives across alll eq.
  - If $x < \beta$, Alice's counteroffer is to offer a menu of mechanisms

$$\left\{ m^*_{1-\delta(1-\beta)+\varepsilon}(G) : \text{for all beliefs } G \right\}.$$

  - Bob will accept it because it improves his payoff, no matter what is the posterior belief G.
  - If ε is small enough, because $x < \beta$, any choice of Bob will lead to a strict improvement for at least some type of Alice.

Proof

Role of menus

- Menus help with screening problem
- menus of menus help with signaling problem (inscrutability), and
- responding to belief threats.

Comments

1. Neutral solution
2. Coasian bargaining
3. Renegotiation
4. Other bargaining environments
5. Two-sided incomplete information

Comments

Neutral solution

- Axiomatic bargaining: Harsanyi and Selten (72), Myerson (84)
  - incentive compatible mechanisms,
- (Myerson 84) - neutral solution as a minimal set of incentive compatible outcomes that satisfies three axioms
  - probability invariance
  - extension axiom,
  - random-dictatorship (with simple bargaining problems .
- In practice, equal sharing of virtual valuations.

Comments

Neutral solution

- Here: assume that $\beta = 1/2$.

Theorem

Suppose that

$$(u - v) f(u) - (1 - F(u))$$

is strictly increasing in u. Then, equal likelihood of "property rights" mechanism is the unique neutral solution.

Comments

Commitment and Coasian bargaining

- Coasian bargaining and dynamic mechanism design without commitment: Skreta (06), Liu et al (19), Doval, Skreta (21),
  - only uninformed party makes offers.
- As in that literature,
  - players cannot *unilaterally* commit to future offers,
  - players are committed to an offer for the period in which the offer is made,
  - once the offer is accepted, it must be implemented.
- But, mechanisms may generate ex post inefficient allocation,
  - players have also access to a large(-r) space of mechanisms,
  - applications: bargaining over protocol, bargaining without common knowledge of surplus

Comments

Commitment and Coasian bargaining

- When $\beta = 0$, Bob is the single proposer, the unique PBE is that Bob proposes optimal selling mechanism: sell at price $p^* > v$, which is accepted.
  - that's unlike Coasian bargaining, where Bob would sell at v:
  - in the Coasian bargaining, if offer is rejected, Bob cannot stop himself from learning that it is rejected,
  - here, rejection does not reveal any information,
- The ability of players to commit to the mechanism once accepted is not crucial - see next!

Comments

Renegotiation

- Multiple ways of introducing renegotiation.
- Suppose that both Alice and Bob need both to agree to renegotiate:
  - after mechanism is accepted, and implemented, one of them may propose "Do you want to renegotiate?"
  - and if the other says "yes", the bargaining game is restarted,
  - so, previous agreement is not a "starting point" for renegotiation (unlike Strulovici 17).
- The lower bound on Bob's payoffs (i.e., β-random property rights) remains the same.

Comments

Renegotiation

- The lower bound on Bob's payoffs remains the same.
  - renegotiation leads to the possibility that players make sub-optimal choice in the counter-offered mechanism, because they anticipate renegotiation,
  - but the argument goes through:
  - the key is that Bob's counteroffer is a menu and Alice controls the payoffs.
- This does not work for Alice:
  - Alice's counteroffer is a menu of menus, and Bob does not control the payoffs.

Comments

Heterogeneous pie

- Heterogeneous pie: chocolate and strawberry part
- $X = \left\{ (a_c, a_s) \in [0, 1]^2 \right\}$ - divisions of the pie
  - Alice's utility: $ua_c + (1 - u)\, a_s$,
  - Bob's utility: $v\,(1 - a_c) + (1 - v)\,(1 - a_s)$.

Theorem

If $\beta = \frac{1}{2}$ and $\delta \to 1$, the PBE outcomes converge to optimal Bob's mechanism st. each type of Alice receiving her complete information payoffs.

Comments

Other bargaining environments

- More generally, redefine
  - the space of allocations X,
  - preference types,
- apply the same methodology.
- General result: Under one-sided incomplete information, each player i will receive at least β_i fraction of their best allocation.

Comments

Two-sided incomplete information

- Suppose that two players can have two types $u_l < u_h$.
  - beliefs $F_i \in \Delta\{u_l, u_h\}$,
- $\beta_A + \beta_B = 1$ proposer probabilities:
- β-random property right mechanism: with prob. β_i, player i gets the good and may offer to sell it at price $p = u_h$.
  - this mechanism is ex post efficient.

Comments

Two-sided incomplete information

Theorem

Suppose $\mathcal{M}$ contains all α-random property rights mechanisms for all $\alpha \in [0, 1]$.
Then, in the unique equilibrium, the expected payoffs are as if β-random property rights mechanism is implemented.

Conclusion

- A model of bargaining with incomplete information and mechanisms as offers
- Main result: unique and continuous equilibrium outcome
  - role of mechanisms in bargaining,
- Proof of a concept that bargaining with mechanisms is possible and useful,
  - relation to axiomatic theory,
  - other environments,
  - two-sided incomplete information,